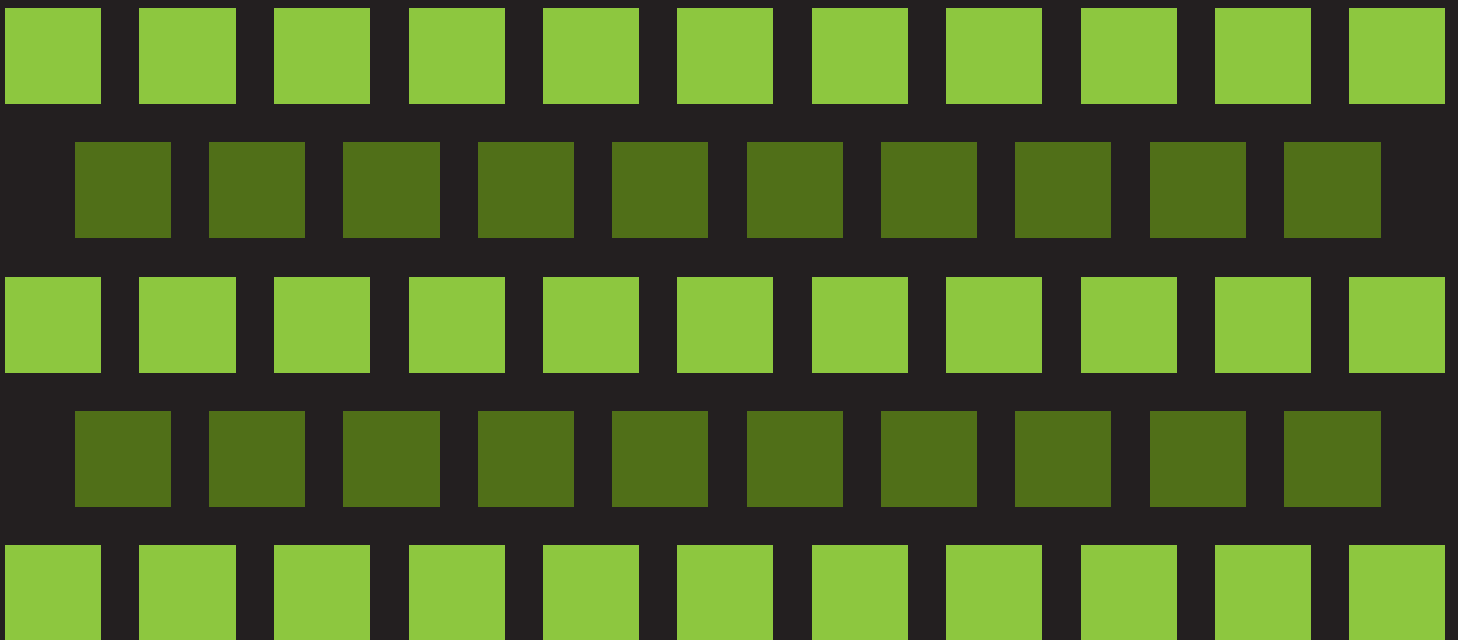


STP-PT-071

# STRESS INTENSITY FACTOR SOLUTIONS FOR INTERNAL CRACKS IN THICK-WALLED CYLINDER VESSELS



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# **STRESS INTENSITY FACTOR SOLUTIONS FOR INTERNAL CRACKS IN THICK-WALLED CYLINDER VESSELS**

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## FOREWORD

Stress intensity K factors were computed for internal cracks in thick-walled cylinders. The 744 cases include a range of geometry ratios and crack size ratios for internal axial and internal circumferential surface cracks, axial internal full-width partial depth axial cracks, and circumferential internal 360° partial depth cracks. These K solutions extend the K factor solutions available in the API 579-1/ASME FFS-1 Annex C tables.

The 3D crack meshes were created using the FEACrack software, and the analyses were run using the Abaqus FEA software. A mesh convergence study examined a variety of mesh settings to confirm that adequate mesh refinement was used to compute the stress intensity values. The post processing included automatic and manual quality checks to confirm the results.

The results are reported as non-dimensional geometry factors that are tabulated in the appendices along with plots of all the result cases. The new solutions can be added to the API 579-1/ASME FFS-1 Annex C tables.

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## **EXECUTIVE SUMMARY**

This report describes the analysis methods and results for internal surface crack stress intensity  $K$  solutions in thick-walled cylinders. The 744 cases include a range of geometry ratios and crack size ratios for internal axial and internal circumferential surface cracks, axial full-width cracks, and 360° circumferential cracks. These  $K$  solutions extend the  $K$  factor solutions available in the 2007 API 579-1/ASME FFS-1 Annex C tables. The results are reported as non-dimensional geometry factors that are tabulated in the appendices along with plots of all the result cases.

## 1 INTRODUCTION

This report computes the stress intensity K factors for internal surface cracks in thick-walled cylinder vessels. The 744 crack analysis cases for this project extend the K factor solutions available in the API 579-1/ASME FFS-1 Annex C tables [1].

The analysis method uses the FEACrack™ [2] software to generate the three-dimensional (3D) crack meshes, described below. The FEACrack software, a commercial product for 3D crack mesh generation and analysis, was originally released in 1998, and was used to create most of the K solutions in Annex C of API 579 [1][4][5]. FEACrack creates complete and ready-to-run Abaqus™ [6] input files, including the syntax to define the J-integral calculation, to allow efficient analysis of many crack cases.

The finite element analysis (FEA) cases are run for each crack mesh using the Abaqus solver. Abaqus also provides the crack front J-integral calculation at the crack front nodes. FEACrack provides automated post processing to help inspect the mesh and crack front J-integral results and tabulate the stress intensity solution factors. FEACrack automatically computes the stress intensity K factor from the J-integral using the elastic material properties, and examines the J-integral path dependence to indicate any issues with a result.

The stress intensity results are reported as non-dimensional geometry G factor values, described in Section 1.2. The result values are tabulated in the appendices. Plots of the results for all the cases examined and for result trends are also shown in the appendices. Each appendix begins with a description of the values or plots within the particular appendix.

### 1.1 Analysis Cases

The crack analysis cases use the geometry ratio  $Y = OD/ID$  and the  $a/l$  crack aspect ratio, where OD is the cylinder outside diameter, ID is the cylinder inside diameter,  $a$  is the crack depth, and  $l$  is the total semi-elliptical crack length. The crack depth ratio  $a/t$  describes the crack depths examined, where  $t$  is the cylinder wall thickness.

The  $Y$  and  $a/l$  ratios are related to the  $t/R_i$  and  $a/c$  ratios used in the API 579 Annex C tables, where  $R_i$  is the cylinder inside radius and  $c$  is the half semi-elliptical surface crack length:  $t/R_i = Y - 1$ ,  $a/l = (a/c)/2$ ,  $2c = l$ . The same  $a/l$  and  $a/t$  ratio values as the Annex C solutions were used so that the new stress intensity solutions can be easily added to the existing solution tables. Both sets of ratios are given in Figure 1-1 through Figure 1-3 below. The  $Y = 2$  ratio ( $t/R_i = 1$ ) overlaps the current solutions so that the new results can be compared to show continuity of values. The new solutions extend the  $Y$  ratio to 4 ( $t/R_i = 3$ ) for the thickest cylinder case examined.

Generic values for geometry and loads are used to create the crack meshes, since the final results are given as the non-dimensional geometry G factors.

**Figure 1-1: Y and t/R<sub>i</sub> Ratios to Set the Cylinder Thickness, t**

| case | Y=OD/ID | t/R <sub>i</sub> |
|------|---------|------------------|
| 1    | 2       | 1                |
| 2    | 2.5     | 1.5              |
| 3    | 3       | 2                |
| 4    | 3.5     | 2.5              |
| 5    | 4       | 3                |