

Australian/New Zealand Standard™

**Power law model — Goodness-of-fit
tests and estimation methods**



AS/NZS IEC 61710:2020

This Joint Australian/New Zealand Standard™ was prepared by Joint Technical Committee QR-005, Dependability. It was approved on behalf of the Council of Standards Australia on 23 March 2020 and by the New Zealand Standards Approval Board on 5 February 2020.

This Standard was published on 3 April 2020.

The following are represented on Committee QR-005:

- Asset Management Council (Australia)
- Australian Industry Group
- Department of Defence (Australian Government)
- Engineering New Zealand
- Engineers Australia
- Human Factors and Ergonomics Society of New Zealand
- Institution of Occupational Safety and Health
- National Rail Safety Regulator (Australia)
- National Road Carriers Association (New Zealand)
- New Zealand Institute of Safety Management
- Professionals Australia
- Risk Engineering Society (Australia)
- Risk Management Institute of Australasia
- RiskNZ
- University of New South Wales
- University of Western Australia
- University of Wollongong

This Standard was issued in draft form for comment as DR AS/NZS IEC 61710:2019.

Keeping Standards up-to-date

Ensure you have the latest versions of our publications and keep up-to-date about Amendments, Rulings, Withdrawals, and new projects by visiting:

www.standards.org.au

www.standards.govt.nz

ISBN 978 1 76072 791 8

Australian/New Zealand Standard™

**Power law model — Goodness-of-fit
tests and estimation methods**

First published as AS/NZS IEC 61710:2020.

COPYRIGHT

© IEC 2020 — All rights reserved

© Standards Australia Limited/the Crown in right of New Zealand, administered by the New Zealand Standards Executive 2020

All rights are reserved. No part of this work may be reproduced or copied in any form or by any means, electronic or mechanical, including photocopying, without the written permission of the publisher, unless otherwise permitted under the Copyright Act 1968 (Cth) or the Copyright Act 1994 (New Zealand).

Preface

This Standard was prepared by the Joint Standards Australia/Standards New Zealand Committee QR-005, Dependability.

The objective of this Standard is to specify procedures to estimate the parameters of the power law model, to provide confidence intervals for the failure intensity, to provide prediction intervals for the times to future failures, and to test the goodness-of-fit of the power law model to data from repairable items. It is assumed that the time to failure data have been collected from an item, or some identical items operating under the same conditions (e.g. environment and load).

This Standard is identical with, and has been reproduced from IEC 61710:2013, *Power law model — Goodness-of-fit tests and estimation methods*.

As this document has been reproduced from an International Standard, the following applies:

- (a) In the source text “this International Standard” should read “this Australian/New Zealand Standard”.
- (b) A full point substitutes for a comma when referring to a decimal marker.

Australian or Australian/New Zealand Standards that are identical adoptions of international normative references may be used interchangeably. Refer to the online catalogue for information on specific Standards.

The terms “normative” and “informative” are used in Standards to define the application of the appendices or annexes to which they apply. A “normative” appendix or annex is an integral part of a Standard, whereas an “informative” appendix or annex is only for information and guidance.

NOTES

CONTENTS

FOREWORD.....	5
INTRODUCTION.....	7
1 Scope.....	8
2 Normative references.....	8
3 Terms and definitions.....	8
4 Symbols and abbreviations.....	8
5 Power law model.....	9
6 Data requirements.....	10
6.1 General.....	10
6.1.1 Case 1 – Time data for every relevant failure for one or more copies from the same population.....	10
6.1.2 Case 1a) – One repairable item.....	10
6.1.3 Case 1b) – Multiple items of the same kind of repairable item observed for the same length of time.....	11
6.1.4 Case 1c) – Multiple repairable items of the same kind observed for different lengths of time.....	11
6.2 Case 2 – Time data for groups of relevant failures for one or more repairable items from the same population.....	12
6.3 Case 3 – Time data for every relevant failure for more than one repairable item from different populations.....	12
7 Statistical estimation and test procedures.....	13
7.1 Overview.....	13
7.2 Point estimation.....	13
7.2.1 Case 1a) and 1b) – Time data for every relevant failure.....	13
7.2.2 Case 1c) – Time data for every relevant failure.....	14
7.2.3 Case 2 – Time data for groups of relevant failures.....	15
7.3 Goodness-of-fit tests.....	16
7.3.1 Case 1 – Time data for every relevant failure.....	16
7.3.2 Case 2 – Time data for groups of relevant failures.....	17
7.4 Confidence intervals for the shape parameter.....	18
7.4.1 Case 1 – Time data for every relevant failure.....	18
7.4.2 Case 2 – Time data for groups of relevant failures.....	19
7.5 Confidence intervals for the failure intensity.....	20
7.5.1 Case 1 – Time data for every relevant failure.....	20
7.5.2 Case 2 – Time data for groups of relevant failures.....	20
7.6 Prediction intervals for the length of time to future failures of a single item.....	21
7.6.1 Prediction interval for length of time to next failure for case 1 – Time data for every relevant failure.....	21
7.6.2 Prediction interval for length of time to <i>R</i> th future failure for case 1 – Time data for every relevant failure.....	22
7.7 Test for the equality of the shape parameters $\beta_1, \beta_2, \dots, \beta_k$	23
7.7.1 Case 3 – Time data for every relevant failure for two items from different populations.....	23
7.7.2 Case 3 – Time data for every relevant failure for three or more items from different populations.....	24
Annex A (informative) The power law model – Background information.....	30
Annex B (informative) Numerical examples.....	31

Annex C (informative) Bayesian estimation for the power law model	41
Bibliography	56
Figure 1 – One repairable item	10
Figure 2 – Multiple items of the same kind of repairable item observed for same length of time	11
Figure 3 – Multiple repairable items of the same kind observed for different lengths of time	12
Figure B.1 – Accumulated number of failures against accumulated time for software system	32
Figure B.2 – Expected against observed accumulated times to failure for software system	32
Figure B.3 – Accumulated number of failures against accumulated time for five copies of a system	35
Figure B.4 – Accumulated number of failures against accumulated time for an OEM product from vendors A and B	37
Figure B.5 – Accumulated number of failures against time for generators	38
Figure B.6 – Expected against observed accumulated number of failures for generators	39
Figure C.1 – Plot of fitted Gamma prior (6,7956, 0,0448) for the shape parameter of the power law model	47
Figure C.2 – Plot of fitted Gamma prior (17,756 6, 1447,408) for the expected number of failures parameter of the power law model	47
Figure C.3 – Subjective distribution of number of failures	51
Figure C.4 – Plot of the posterior probability distribution for the number of future failures, M	54
Figure C.5 – Plot of the posterior cumulative distribution for the number of future failures, M	55
Table 1 – Critical values for Cramer-von-Mises goodness-of-fit test at 10 % level of significance	25
Table 2 – Fractiles of the Chi-square distribution	26
Table 3 – Multipliers for two-sided 90 % confidence intervals for intensity function for time terminated data	27
Table 4 – Multipliers for two-sided 90 % confidence intervals for intensity function for failure terminated data	28
Table 5 – 0,95 fractiles of the F distribution	29
Table B.1 – All relevant failures and accumulated times for software system	31
Table B.2 – Calculation of expected accumulated times to failure for Figure B.2	33
Table B.3 – Accumulated times for all relevant failures for five copies of a system (labelled A, B, C, D, E)	34
Table B.4 – Combined accumulated times for multiple items of the same kind of a system	34
Table B.5 – Accumulated operating hours to failure for OEM product from vendors A and B	36
Table B.6 – Grouped failure data for generators	38
Table B.7 – Calculation of expected numbers of failures for Figure B.6	40
Table C.1 – Strengths and weakness of classical and Bayesian estimation	42

Table C.2 – Grid for eliciting subjective distribution for shape parameter β 46

Table C.3 – Grid for eliciting subjective distribution for expected number of failures
parameter η 46

Table C.4 – Comparison of fitted Gamma and subjective distribution for shape
parameter β 48

Table C.5 – Comparison of fitted Gamma and subjective distribution for expected
number of failures by time $T = 20\,000$ h parameter η 48

Table C.6 – Times to failure data collected on system test..... 49

Table C.7 – Summary of estimates of power law model parameters 50

Table C.8 – Time to failure data for operational system 53

INTERNATIONAL ELECTROTECHNICAL COMMISSION

**POWER LAW MODEL –
GOODNESS-OF-FIT TESTS
AND ESTIMATION METHODS**

FOREWORD

- 1) The International Electrotechnical Commission (IEC) is a worldwide organization for standardization comprising all national electrotechnical committees (IEC National Committees). The object of IEC is to promote international co-operation on all questions concerning standardization in the electrical and electronic fields. To this end and in addition to other activities, IEC publishes International Standards, Technical Specifications, Technical Reports, Publicly Available Specifications (PAS) and Guides (hereafter referred to as “IEC Publication(s)”). Their preparation is entrusted to technical committees; any IEC National Committee interested in the subject dealt with may participate in this preparatory work. International, governmental and non-governmental organizations liaising with the IEC also participate in this preparation. IEC collaborates closely with the International Organization for Standardization (ISO) in accordance with conditions determined by agreement between the two organizations.
- 2) The formal decisions or agreements of IEC on technical matters express, as nearly as possible, an international consensus of opinion on the relevant subjects since each technical committee has representation from all interested IEC National Committees.
- 3) IEC Publications have the form of recommendations for international use and are accepted by IEC National Committees in that sense. While all reasonable efforts are made to ensure that the technical content of IEC Publications is accurate, IEC cannot be held responsible for the way in which they are used or for any misinterpretation by any end user.
- 4) In order to promote international uniformity, IEC National Committees undertake to apply IEC Publications transparently to the maximum extent possible in their national and regional publications. Any divergence between any IEC Publication and the corresponding national or regional publication shall be clearly indicated in the latter.
- 5) IEC itself does not provide any attestation of conformity. Independent certification bodies provide conformity assessment services and, in some areas, access to IEC marks of conformity. IEC is not responsible for any services carried out by independent certification bodies.
- 6) All users should ensure that they have the latest edition of this publication.
- 7) No liability shall attach to IEC or its directors, employees, servants or agents including individual experts and members of its technical committees and IEC National Committees for any personal injury, property damage or other damage of any nature whatsoever, whether direct or indirect, or for costs (including legal fees) and expenses arising out of the publication, use of, or reliance upon, this IEC Publication or any other IEC Publications.
- 8) Attention is drawn to the Normative references cited in this publication. Use of the referenced publications is indispensable for the correct application of this publication.
- 9) Attention is drawn to the possibility that some of the elements of this IEC Publication may be the subject of patent rights. IEC shall not be held responsible for identifying any or all such patent rights.

International Standard IEC 61710 has been prepared by IEC technical committee 56: Dependability.

This second edition cancels and replaces the first edition, published in 2000, and constitutes a technical revision.

The main changes with respect to the previous edition are listed below:

- the inclusion of an additional Annex C on Bayesian estimation for the power law model.

The text of this standard is based on the following documents:

FDIS	Report on voting
56/1500/FDIS	56/1508/RVD

Full information on the voting for the approval of this standard can be found in the report on voting indicated in the above table.

This publication has been drafted in accordance with the ISO/IEC Directives, Part 2.

The committee has decided that the contents of this publication will remain unchanged until the stability date indicated on the IEC web site under "<http://webstore.iec.ch>" in the data related to the specific publication. At this date, the publication will be

- reconfirmed,
- withdrawn,
- replaced by a revised edition, or
- amended.

IMPORTANT – The 'colour inside' logo on the cover page of this publication indicates that it contains colours which are considered to be useful for the correct understanding of its contents. Users should therefore print this document using a colour printer.

INTRODUCTION

This International Standard describes the power law model and gives step-by-step directions for its use. There are various models for describing the reliability of repairable items, the power law model being one of the most widely used. This standard provides procedures to estimate the parameters of the power law model and to test the goodness-of-fit of the power law model to data, to provide confidence intervals for the failure intensity and prediction intervals for the length of time to future failures. An input is required consisting of a data set of times at which relevant failures occurred, or were observed, for a repairable item or a set of copies of the same item, and the time at which observation of the item was terminated, if different from the time of final failure. All output results correspond to the item type under consideration.

Some of the procedures can require computer programs, but these are not unduly complex. This standard presents algorithms from which computer programs should be easy to construct.

POWER LAW MODEL – GOODNESS-OF-FIT TESTS AND ESTIMATION METHODS

1 Scope

This International Standard specifies procedures to estimate the parameters of the power law model, to provide confidence intervals for the failure intensity, to provide prediction intervals for the times to future failures, and to test the goodness-of-fit of the power law model to data from repairable items. It is assumed that the time to failure data have been collected from an item, or some identical items operating under the same conditions (e.g. environment and load).

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-191:1990, *International Electrotechnical Vocabulary (IEV) – Chapter 191: Dependability and quality of service*

3 Terms and definitions

For the purposes of this document, the terms and definitions of IEC 60050-191 apply.

4 Symbols and abbreviations

The following symbols and abbreviations apply:

β	shape parameter of the power law model
$\hat{\beta}$	estimated shape parameter of the power law model
β_{LB}, β_{UB}	lower, upper confidence limits for β
C^2	Cramer-von-Mises goodness-of-fit test statistic
$C_{1-\gamma}^2(M)$	critical value for the Cramer-von-Mises goodness-of-fit test statistic at γ level of significance
χ^2	Chi-square goodness-of-fit test statistic
$\chi_{\gamma}^2(\nu)$	γ th fractile of the χ^2 distribution with ν degrees of freedom
d	number of intervals for groups of failures
$E[N(t)]$	expected accumulated number of failures up to time t
$E[t_j]$	expected accumulated time to j th failure